

Supplementary Materials

Literature search

We collected English language publications containing studies investigating insight in problem solving through a search of the PsycINFO, PsycARTICLES, Scopus, Social Sciences Citation Index, and Web of Science databases, as well as searching Google Scholar and ProQuest Digital Dissertations for doctoral dissertations, unpublished articles, and conference articles, using the key words insight*, insight problem*, problem solving, aha*, and eureka moment.

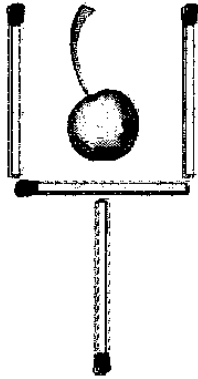
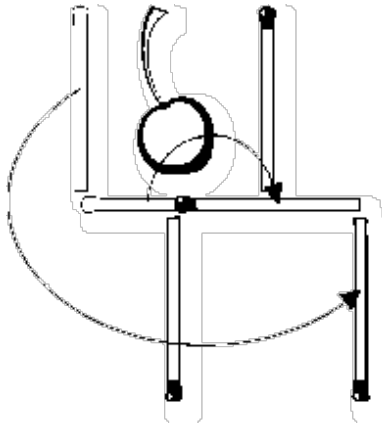
Table 1.

Table of problems in descending order of times found in the literature

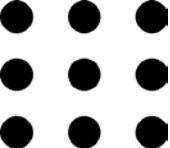
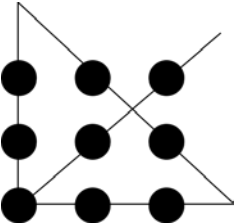
<u>Insight problems</u>		<u>Non-insight problems</u>	
Problem name	Papers found in	Problem name	Papers found in
Triangle of coins	32	Cards	15
Lilies	19	Tower of Hanoi	10
Antique Coin	18	Dinner	7
Nine-dot problem	16	Police	7
Necklace problem	15	Anagrams	6
Socks	13	Heavy/Light coins	5
Candle Problem	12	Math problems	5
Horse trading problem	13	Tower of London	5
Pig Pen	11	Bachelor	4
		Matchstick	
Marrying man	10	arithmetic (ST)	5
Prisoner	10	ATM Bob	3
2-string	9	Camping trip	3
Pyramid dollar	8	Hobbit and Orcs	3
Radiation problem	8	Calendar	3
10 tree	7	Raven's (RAPM)	3
Card hole	6	Trace	3
Earth	5	Age	3
Farm	5	Flower problem	3
Football	5	Puzzling puzzle	2
Light switch	5		
Ping-pong ball	5		
t-puzzle	5		
Train	5		
8 coins	4		
Basketball win	4		
Ladder	5		

Table 2.

Classic insight problems and their solutions.

Name	Problem description	Solution
2-string	The two strings dangling from the ceiling are too far apart for the decorator to grasp the both at the same time; how can he use the objects in the room to help him tie them together?	Use a heavy objects to weight the string and swing them both to get them to reach one another in the middle
10 tree	How can you plant 10 trees in five rows with four trees in each row?	Make a star (5 lines of 4 dots)
Candle problem	You have a candle, some matches, and a box of tacks. How can you support the candle on the wall?	Use the tacks to attach the match box to the wall
Card hole	Describe how to cut a hole in a 3 X 5 in. card that is big enough for you to put your head through.	Spiral design in the middle of the card
Cherry	Move 2 matches so cherry is outside the glass	
		
		
Coin problem	A dealer of antique coins received an offer to buy a beautiful bronze coin by an unknown man. The coin had an emperor's head on one side and the date 544 B.C. stamped on the other side. The dealer examined the coin, but instead of buying it, he called the police to arrest the man. What made him realise that the coin was fake?	BBC is a term used after Christ, not before

Name	Problem description	Solution
Earth hole	How much earth is there in a hole 3 ft by 3 ft by 3 ft	None
Fan	Joe Fan has no psychic powers but he can tell you the score of any football game before it starts. How?	The score of any game before it starts is 0
Horse trading problem	A man buys a horse for \$60, sells it for \$70, buys it back for \$80 and sells it finally for \$90. How much has he made?	\$20
Ladder	Mr. Hardy was washing windows on a high-rise office building when he slipped and fell off a 60-foot ladder onto the concrete sidewalk below. Incredibly, he did not injure himself in any way. How is this possible?	He was not high off the ground
Light switch	A young boy turned of the light in his bedroom and managed to get into bed before the room was dark. If the bed is ten feet from the light switch and he used no wires, strings or other contraptions to turn off the light, how did he do it?	It was daytime
Lilies	Water lilies double in area every 24 hours. At the beginning of summer there is one water lily on the lake. It takes 60 days for the lake to become completely covered with water lilies. On which day is the lake half covered?	59
Marrying Man	A man in a small town married 20 different women of the same town. All are still living and he never divorced. Polygamy is unlawful but he has broken no law. How can this be?	The man is a priest
Necklace problem	You are given four separate sets of chain links (A, B, C, and D), as shown in the Initial State. Each set is composed of 3 links which are closed at the beginning of the problem. It costs 2 cents to open a link and 3 cents to close a link. Your goal is to form a complete closed necklace, as shown in the Goal State, using sets A, B, C, and D, at a cost of no more than 15 cents.	Break one of the three initial state chains into 3, and use those to connect the other three

Name	Problem description	Solution
Nine-dot problem	<p>The task is to connect all nine dots with four straight lines, drawn without lifting pencil from paper or retracing a line.</p> 	
Pig Pen	<p>Nine pigs are kept in a square pen. Build two more square enclosures that would put each pig in a pen by itself</p>	Square within diamond
Ping-pong ball	<p>How do you throw a ping-pong ball so that it will travel a short distance, come to a dead stop and then reverse itself?</p>	Throw the ball up in the air
Pound coins	<p>Which would be worth more? A pound of \$10 pure gold coins or a half a pound of \$20 pure gold coins; or would they be worth the same amount?</p>	Gold is worth more than pounds
Prisoner	<p>A prisoner was attempting to escape from a tower. He found in his cell a rope that was half long enough to permit him to reach the ground safely. He divided the rope in half and tied the two parts together and escaped. How?</p>	Half lengthwise
Pyramid dollar	<p>A giant inverted steel pyramid is perfectly balanced on its point. Any movement of the pyramid will cause it to topple over. Underneath the pyramid is a \$100 bill. How would you remove the bill without disturbing the pyramid?</p>	Burn or tear the note
Socks problem	<p>If you have black socks and brown socks in your drawer, mixed in a ratio of 4 to 3, how many socks will you have to take out to make sure that you have a pair the same color?</p>	3


Name	Problem description	Solution
Radiation problem	Given a human being with an inoperable stomach tumour, and lasers which destroy organic tissue at sufficient intensity, how can one cure the person with these lasers and, at the same time, avoid harming the healthy tissue that surrounds the tumour?	Angle multiple lasers that are at a low intensity, so that they meet at the tumour, but where they do not meet have a low enough intensity that the stomach tissue remains undamaged.
Train	At 7am, a train moving 90kmh leaves Montreal heading for Toronto. At 8am, a train running 100kmh leaves Toronto heading from Montreal. Which train will be closer to Montreal when they meet?	They meet at the same spot, so the same distance from Montreal
Triangle of coins problem	Moving only 3 coins, make the triangle point downwards.	

Table 3.

Classic non-insight problems and their solutions

Name	Problem description	Solution
Age	Bob's father is 3 times as old as Bob. They were both born in October. 4 years ago, he was 4 times older. How old are Bob and his father?	<p>Bob's father is 3 times as old as Bob. Bob is now 12, his father is 36.</p> $3 \times 12 = 36$ <p>They were both born in October. 4 years ago, he was 4 times older?</p> <p>Four years ago Bob was 8, his father was 32.</p> $4 \times 8 = 32$

Bachelor Five bachelors, Andy, Bill, Carl, Dave, and Eric, go out together to eat five evening meals (Fish, Pizza, Steak, Tacos, and Thai) on Monday through Friday. It was understood that Eric would miss Friday's meal due to an out of town wedding. Each bachelor served as the host at a restaurant of his choice on a different night. The following information is known: Carl hosted the group on Wednesday. The fellows ate at a Thai restaurant on Friday. Bill, who detests fish, volunteered to be the first host. Dave selected a steak house for the night before one of the fellows hosted everyone at a raucous pizza parlour. Which bachelor hosted the group each night and what food did he select?

DAY	HOST	FOOD
Mon	Bill	Tacos
Tue	Dave	Steak
Wed	Carl	Pizza
Thu	Eric	Fish
Fri	Andy	Thai

Card problem Three cards from an ordinary deck are lying on a table, face down. The following information (for some peculiar reason) is known about those three cards (all the information below refers to the same three cards):
 To the left of a queen there is a jack
 To the left of a spade there is a diamond
 To the right of a heart there is a king
 To the right of a king there is a spade
 Can you assign the proper suit to each picture card?

Jack of hearts, king of diamonds, queen of spades

Couples Three couples went together to a party. One woman was dressed in red, one in green, and one in blue. Each man was wearing one of these colours. When all three couples were dancing, the man in red was dancing with the woman in blue. Isn't it funny Christine, not one of us is dancing with a partner dressed in the same colour; Think about the man who is dancing with the woman in red. What colour is he wearing?

Green. Since the man in blue is dancing with the woman in red the man in green has the choice of either the woman in red or green (he cannot dance with green so he is dancing with the woman in red).

Dinner Mary won't eat fish or spinach, Sally won't eat fish or green beans, Steve won't eat shrimp or potatoes, Alice won't eat beef or tomatoes, and Jim won't eat fish or tomatoes. If you are willing to give such a bunch of fussy eaters a dinner party, which items from the following list can you serve: green beans, creamed codfish, roast beef, roast chicken, celery, and lettuce.

Roast chicken, celery, and lettuce

Name	Problem description	Solution
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Heavy/ Light coins	Given 4 coins, of which two are slightly heavy and two slightly light, but which look and feel identical, how could you find out which are which in two weighings on a balance scale?	Begin by placing one coin on each side of the scale. If they do not balance, then you have already identified one heavy and one light coin. Repeating the procedure with the remaining two coins will identify the other light and heavy coins. If the initial two coins balance, simply remove one of the coins and replace it with one of the remaining coins. This weighing will provide the remaining information needed to determine which coins are heavy and which are light.
Hobbit and Orcs	Given a boat that can only take two creatures, how can you get three hobbits and three orcs across a river in such a way that the hobbits are never outnumbered by the orcs on either side?	<p>Trip 1a: 2 Orcs across -- that leaves 3 Hobbits and 1 Orc on Side A</p> <p>Trip 1b: Send 1 Orc back with the boat. -- that puts 1 Orc on Side B</p> <p>Trip 2a: Send 2 Orcs across -- that leaves 3 Hobbits on side A</p> <p>Trip 2b: Send 1 Orc back with the boat -- that puts 2 Orcs on Side B</p> <p>Trip 3a: Get Orc out of boat and send 2 hobbits over -- that leaves 1 Hobbit and 1 Orc on Side A</p> <p>Trip 3b: Hobbits get out. One orc brings back boat -- that puts 2 Hobbits and 1 Orc on Side B</p> <p>Trip 4a: Send 1 Orc and 1 Hobbit -- that leaves 1 Orc on Side A</p> <p>Trip 4b: Send 1 Orc back -- that puts 3 Hobbits and 1 Orc on Side B</p> <p>Trip 5: The 2 remaining Orcs cross.</p>
Police	<p>The police were convinced that either A, B, C, or D had committed a crime. Each of the suspects, in turn, made a statement, but only one of the four statements was true.</p> <p>A said, "I didn't do it."</p> <p>B said, "A is lying."</p> <p>C said, "B is lying."</p> <p>D said, "B did it."</p> <p>Who is telling the truth? and</p> <p>Who committed the crime?</p>	B is telling the truth, and A committed the crime
Puzzling puzzle	If the puzzle you solved before you solved this one was harder than the puzzle you solved after you solved the puzzle you solved before you solved this one, was the puzzle you solved before you solved this one harder than this one?	Answer is yes. (There are only 2 puzzles being spoken of: this one, and the one before this one.)
Plan day/	Next week, I am going to have lunch with my friend, visit the new art gallery,	Friday

Weekly planner	go to the Social Security office, and have my teeth checked at the dentist. My friend cannot meet me on Wednesday, the Social Security office is closed weekends; the dentist has office hours only on Tuesday, Friday, and Saturday; the art gallery is closed Tuesday, Thursday, and weekends. On which day can I do everything I have planned?	
Water jug problem	Given a source of unlimited water and 4 containers of different capacities – 99, 14, 25 and 11 – obtain exactly 86 L of water.	simplest: $(25 \times 3) + 11$
Wolf and Chicken	A man has to get a fox, a chicken, and a sack of corn across a river. He has a rowboat, and it can only carry him and one other thing. If the fox and the chicken are left together, the fox will eat the chicken. If the chicken and the corn are left together, the chicken will eat the corn. How does the man do it?	The man and the chicken cross the river, (the fox and corn are safe together), he leaves the chicken on the other side and goes back across. The man then takes the fox across the river, and since he can't leave the fox and chicken together, he brings the chicken back. Again, since the chicken and corn can't be left together, he leaves the chicken and he takes the corn across and leaves it with the fox. He then returns to pick up the chicken and heads across the river one last time.

Examples of problem types from Table 1

Arithmetic problem from (Ash & Wiley, 2008)

$$\underline{3Z * 3 = 27}$$

$$\underline{2C - 9 = Z}$$

$$P - C = 2D$$

$$\underline{5Z - 11 = M}$$

$$2X = 56 + A$$

$$\underline{8M - C = Y}$$

$$3Y + 14 = X$$

Gear rotation pathways (Stephen et al., 2009)

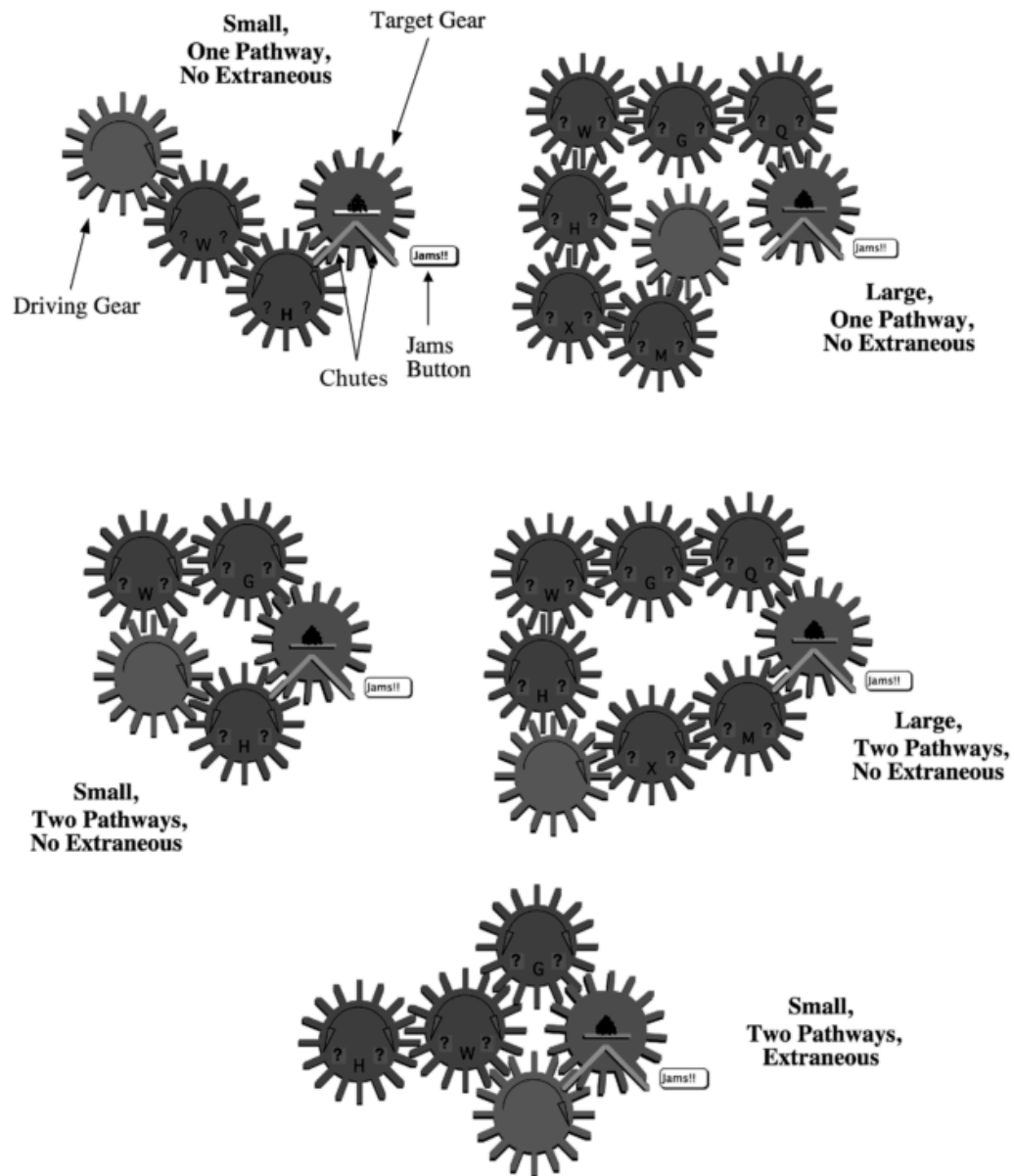


Figure 1. Examples of gear-system problems. The gear systems varied along three dimensions: size (small, 4 or 5 gears; large, 7 or 8 gears), number of pathways (one or two), and whether an extraneous gear was present. Extraneous gears were not part of the causal pathway from the driving gear to the target gear. Gear systems with two pathways had the po-

Doodles (Nishimoto et al., 2010)

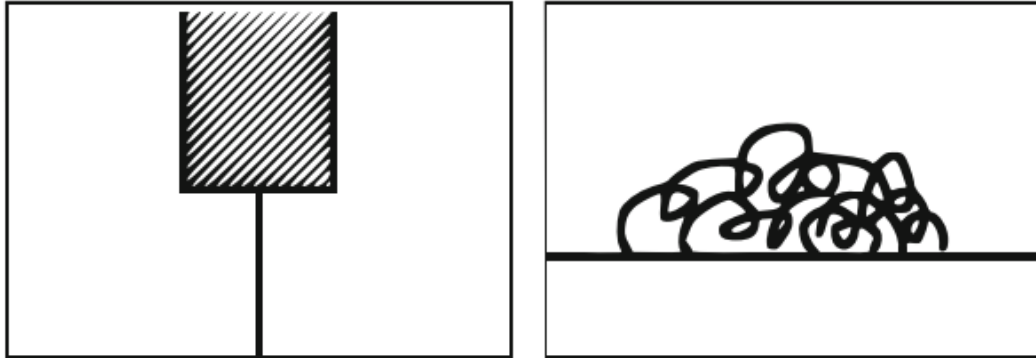
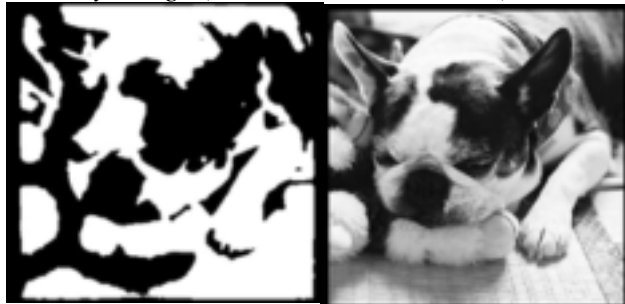
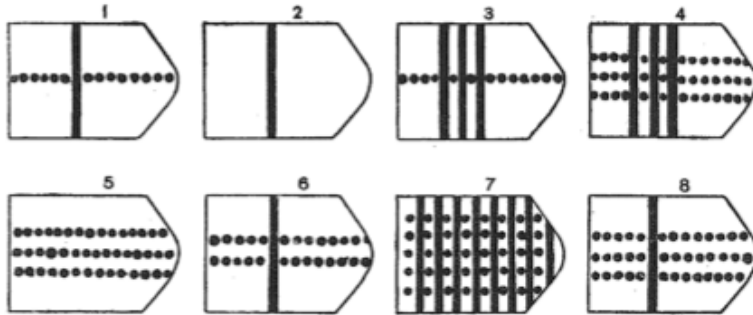
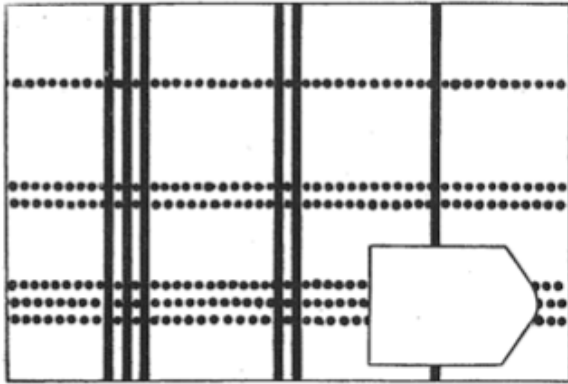


Figure 1. A pair of doodles with the label “Piano keys and the hair of Beethoven while he is playing the piano.”

Mooney image (Kizilirmak et al., 2015)



Raven's problem (Raven, 1985)



Correlation matrices for Figure 1, main text.

Supplementary Table 1.

Pearson Correlations

		Insight	Non-insight	Raven's	CRAs	Anagrams
Classic insight	Pearson's r	—	0.466***	0.357***	0.378***	0.222*
	p-value	—	< .001	< .001	< .001	0.026
Classic non-insight	Pearson's r		—	0.56***	0.251*	0.181
	p-value		—	< .001	0.013	0.392
Raven's	Pearson's r			—	0.062	-0.086
	p-value			—	0.541	0.392
CRAs	Pearson's r				—	0.508***
	p-value				—	< .001
Anagrams	Pearson's r					—
	p-value					—

* p < .05, ** p < .01, *** p < .001

Supplementary Table 2.

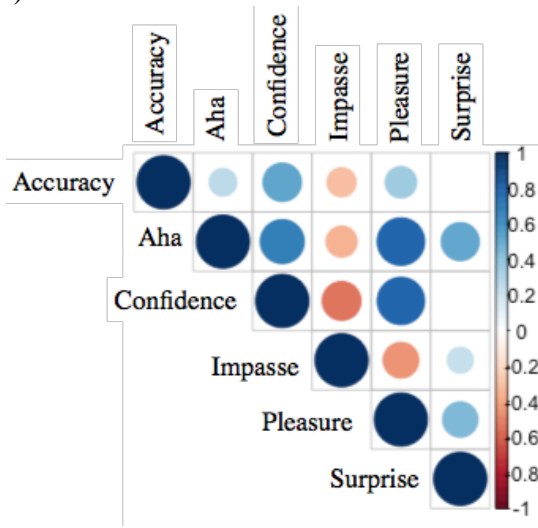
Person correlations between ratings of aha across problem types

Pearson Correlations

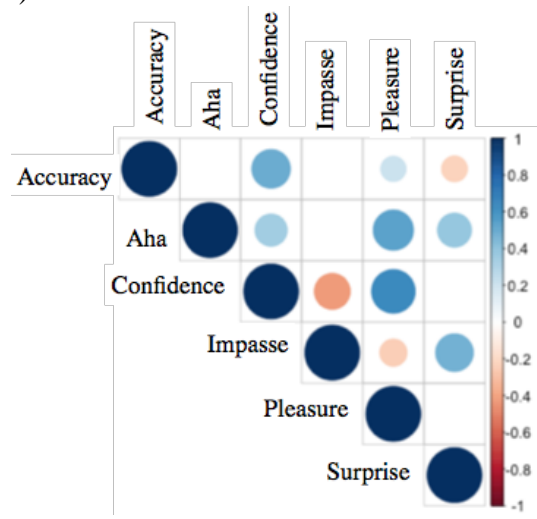
		Insight	Non-insight	Raven's	CRAs	Anagrams
Classic insight	Pearson's r	—	0.774***	0.700***	0.462***	0.408***
	p-value	—	< .001	< .001	< .001	< .001
Classic non-insight	Pearson's r		—	0.733***	0.405***	0.449***
	p-value		—	< .001	< .001	< .001
Raven's	Pearson's r			—	0.358***	0.331***
	p-value			—	< .001	< .001
CRAs	Pearson's r				—	0.618***
	p-value				—	< .001
Anagrams	Pearson's r					—
	p-value					—

* p < .05, ** p < .01, *** p < .001

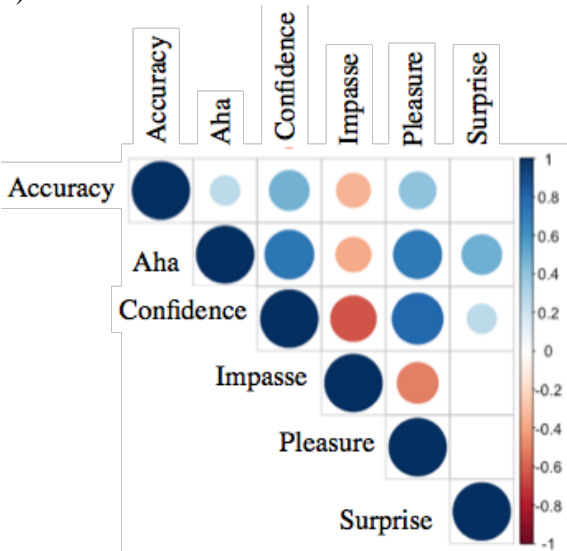
a)



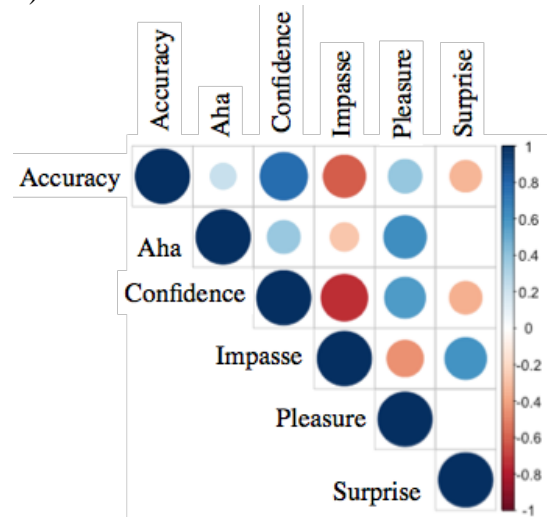
b)



c)



d)



Supplementary Figure 1. Correlation plots between accuracy and aha across problem types: a) Classic insight problems, b) Classic non-insight problems, c) Compound remote associates, d) Anagrams. Size of the circle and saturation of colour determine the strength of the correlation; the colour determined the direction of the relationship, with positive being blue. Non-significant correlations have been inhibited.

Vignette regarding insight affect from Danek, Fraps, von Müller, Grothe and Öllinger (2014):

We would also like to know whether you experienced a feeling of insight when you solve each task: A feeling of insight is a kind of “Aha!” characterised by suddenness and obviousness (and often relief!) - like a revelation. You are relatively confident that your solution is correct without having to check it. In contrast, you experienced no Aha! if the solution occurs to you slowly and stepwise. As an example, imagine a light bulb that is switched on all at once in contrast to slowly dimming it up. We ask for your subjective rating whether it felt like an Aha! experience or not, there is no right or wrong answer. Just follow your intuition.

